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MULTIDIMENSIONAL NONLINEAR LANGMUIR WAVES

P. K. C. WANG

UCLA • SCHOOL OF ENGINEERING AND APPLIED SCIENCE

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Nonlinear Langmuir waves in a plasma governed by the dimensionless equations: $i\partial E/\partial t = -\nabla^2 E + nE$, $\partial^2 n/\partial t^2 = \nabla^2(n + g(|E|^2))$ are studied, where E is the complex amplitude of the high-frequency electric field; n is the low frequency perturbation in the ion density from its constant equilibrium value; and g is a given function of $|E|^2$. General conditions for the existence or nonexistence of a class of multidimensional solitary-wave and nonlinear periodic travelling-wave solutions in the form $E(t,x) = h(k\cdot x - vt)$ and $n(t,x) = s(k\cdot x - vt)$ are established. The results are applied to the special cases: (i) $g(|E|^2) =$

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$|E|^2$ corresponding to the usual pondermotive force, and (ii) $g(|E|^2) = K[1 - \exp(-|E|^2)]$, K is a positive constant, representing ion density saturation.

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Nonlinear Langmuir waves in a plasma governed by the dimensionless equations: $i\partial E/\partial t = -\nabla^2 E + nE$, $\partial^2 n/\partial t^2 = \nabla^2(n + g(|E|^2))$ are studied, where E is the complex amplitude of the high-frequency electric field; n is the low frequency perturbation in the ion density from its constant equilibrium value; and g is a given function of $|E|^2$. General conditions for the existence or nonexistence of a class of multidimensional solitary-wave and nonlinear periodic travelling-wave solutions in the form $E(t,x) = h(k\cdot x-vt)$ and $n(t,x) = s(k\cdot x-vt)$ are established. The results are applied to the special cases: (i)$g(|E|^2) = |E|^2$ corresponding to the usual pondermotive force, and (ii) $g(|E|^2) = K[1-\exp(-|E|^2)]$, K is a positive constant, representing ion density saturation.

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1. INTRODUCTION

The formation, interaction and collapse of nonlinear Langmuir waves in plasmas have been studied extensively in recent years.[1-10] In most of the existing works, attention is focused primarily on the formation and interaction of solitary waves. Exact expressions for these solitary waves for various regimes have been obtained only for the one-dimensional case. Recently, Gibbons et al[10] discussed the possibility of existence of solitary Langmuir waves for higher dimensions. was discussed In this study, ~~we~~ obtain conditions for the existence or nonexistence of multidimensional, nonlinear Langmuir travelling waves, including the periodic and the usual solitary waves.

We begin with the following basic equations[1] describing the nonlinear interaction of high-frequency electron oscillations with an ion fluid:

$$\begin{aligned} i\partial E/\partial t &= -\nabla^2 E + nE, \\ \partial^2 n/\partial t^2 &= \nabla^2(n + g(|E|^2)), \end{aligned} \tag{1}$$

where $i = \sqrt{-1}$; $E = (E_1,\ldots,E_N)$ is the complex amplitude of the high-frequency electric field $\mathcal{E}$ given by

$$\mathcal{E}(t,x) = \mathrm{Re}\{E(t,x)\exp(-i\omega_p t)\}; \tag{2}$$

n is a real quantity corresponding to the low-frequency perturbation in the ion density from its constant equilibrium value n_o; and g is a specified real-valued function of $|E|^2$. Here, we have used dimensionless quantities. The units of time t, spatial coordinates $x = (x_1,\ldots,x_N)$, electric field E and density n are, respectively, $3q/(2\alpha\omega_p)$, $(3r_D/2)(q\alpha)^{-\frac{1}{2}}$, $8(q\alpha n_o\pi T/3)$ and $4q\alpha n_o/3$, where α is the electron-ion mass ratio m_e/m_i; $q = T/T_e$; $T = T_e + T_i$; T_e, T_i the electron and ion temperatures respectively; r_D the electron Debye radius, and ω_p the plasma frequency. The function g is

introduced here so as to permit the consideration of a wide class of nonlinear effects such as saturation.

2. TRAVELLING WAVE SOLUTIONS

Let $\mathbb{R}^N$ and $\mathbb{C}^N$ denote the N-dimensional real and complex Euclidean spaces respectively, and $C_m(\mathbb{R};V)$ the space of all m-times continuously differentiable functions defined on $\mathbb{R}$ and taking their values in the vector space V. The norms for $\mathbb{R}^N$ and $\mathbb{C}^N$ are denoted by $\|\cdot\|$ and $|\cdot|$ respectively. The dot notation is used to denote the usual scalar product on $\mathbb{R}^N$ or $\mathbb{C}^N$.

Let k be a specified unit vector in $\mathbb{R}^N$ and v a given real number corresponding to a constant dimensionless velocity. We seek travelling-wave solutions of (1) in the form:

$$\begin{aligned} E(t,x) &= h(k\cdot x - vt), \\ n(t,x) &= s(k\cdot x - vt), \end{aligned} \tag{3}$$

where h and s are undetermined functions in $C_2(\mathbb{R},\mathbb{C}^N)$ and $C_2(\mathbb{R},\mathbb{R})$ respectively. For physical reasons, we shall restrict h and s to functions such that $|h(\xi)|$ and $|s(\xi)|$ are uniformly bounded on $\mathbb{R}$, where $\xi = k\cdot x - vt$. In particular, we shall consider multidimensional solitary-wave solutions which are analogous to those in the one-dimensional case. Here, we require that $|h(\xi)|$ and $s(\xi)$ tend to finite values as $|\xi| \to \infty$.

Substituting (3) into (1) leads directly to the following equations for $h = (h_1,\ldots,h_N)$ and s:

$$-iv\, dh/d\xi + d^2h/d\xi^2 = s(\xi)h(\xi), \tag{4}$$

$$(v^2 - 1)d^2s/d\xi^2 = d^2g(|h|^2)/d\xi^2, \tag{5}$$

where we have adopted the rectangular Cartesian coordinate system.

Equation (5) can be integrated to give

$$(v^2-1)s(\xi) = g(|h(\xi)|^2) + \hat{C}\xi + C, \tag{6}$$

where $\hat{C}$ and C are integration constants. From the boundedness requirement, we set $\hat{C} = 0$. Assuming that $v^2 \neq 1$, we can solve for $s(\xi)$ in (6) and substitute it into (4) to give a complex differential equation for h:

$$d^2h/d\xi^2 - ivdh/d\xi = (v^2-1)^{-1}\{g(|h(\xi)|^2) + C\}h. \tag{7}$$

It is advantageous to rewrite (7) in polar form. Let $h_j(\xi) = A_j(\xi) \exp(i\theta_j(\xi))$, $j = 1,\ldots,N$. Then, we have

$$d^2A_j/d\xi^2 + A_j\theta_j'(\xi)(v-\theta_j'(\xi)) = (v^2-1)^{-1}A_j\{g(\|A\|^2) + C\}, \tag{8}$$

$$d^2\theta_j/d\xi^2 = (v-2\theta_j'(\xi))d(\ln A_j)/d\xi, \quad j = 1,\ldots,N, \tag{9}$$

where $A = (A_1,\ldots,A_N)$, $\|A\| = |h|$ and $\theta_j' = d\theta_j/d\xi$.

Equation (9) can be integrated to give

$$\theta_j'(\xi) = (v-\mu_j A_j^{-2})/2, \tag{10}$$

where $\mu_j = A_j^2(0)(v-2\theta_j'(0))$.

Substituting (10) into (8) leads to the following differential equations for A_j:

$$d^2A_j/d\xi^2 = f(\mu_j,C,A)A_j, \quad j = 1,\ldots,N, \tag{11}$$

where

$$f(\mu_j,C,A) \triangleq (\mu_j^2A_j^{-4} - v^2)/4 + (v^2-1)^{-1}\{g(\|A\|^2) + C\}. \tag{12}$$

Evidently, given $C, \theta_j'(0), A_j(0), A_j'(0)$, $j = 1,\ldots,N$, (11) can be integrated independently. Since f is a function of C and μ_j (depending on

$A_j(0)$), (11) must be solved with initial conditions at $\xi=0$ which are consistent with the $A_j(0)$ in μ_j. Also, only those portions of solutions of (11) with $A(\xi)\geq 0$ (i.e. $A_j(\xi)\geq 0$, $j=1,\ldots,N$) are meaningful here.

We note that (11) can be rewritten in the form:

$$d^2A_j/d\xi^2 = \partial U/\partial A_j, \quad j=1,\ldots,N, \tag{13}$$

where

$$U(A,\mu,C) \triangleq U_1(\|A\|^2,C) - \sum_{j=1}^{N} \mu_j^2/(8A_j^2), \tag{14}$$

$$2U_1(\|A\|^2,C) \triangleq \int_0^{\|A\|^2} \{\kappa g(\eta) + \gamma\}d\eta, \tag{15}$$

$$\kappa = (v^2-1)^{-1}, \qquad \gamma = (v^2-1)^{-1}C - v^2/4 \tag{16}$$

and $\mu = (\mu_1,\ldots,\mu_N)$. A first integral of (13) is given by

$$I(A(\xi),A'(\xi)) \triangleq \|A'(\xi)\|^2 - 2U_1(\|A(\xi)\|^2,C) - \sum_{j=1}^{N} \mu_j^2 A_j^{-2}(\xi)/4 = C_1, \tag{17}$$

where $\|A'(\xi)\|^2 \triangleq \sum_{j=1}^{N} (dA_j(\xi)/d\xi)^2$ and

$$C_1 = \|A'(0)\|^2 - 2U_1(\|A(0)\|^2,C) - \sum_{j=1}^{N} (v - 2\theta_j'(0)). \tag{18}$$

Evidently, if $\mu_j\neq 0$ for some j, then $I(A(\xi),A'(\xi))\to -\infty$ as $\|A'(\xi)\|$ and $\|A(\xi)\|\to 0$. Since C_1 is finite for finite $\|A'(0)\|$, $\|A(0)\|$, C and $\theta_j'(0)$, $j=1,\ldots,N$, therefore there do not exist solutions of (13) or solitary-wave solutions of (7) such that $\|A(\xi)\|$ and $\|A'(\xi)\|\to 0$ as $|\xi|\to\infty$ when $\mu_j\neq 0$ for some j.

In what follows, we shall focus attention on the particular case where $\mu = 0$. Here, we have

$$\theta_j(\xi) = \theta_j(0) + v\xi/2, \quad j=1,\ldots,N \tag{19}$$

as a solution of (10) or (9). Note that $\mu_j = 0$ when $A_j(0)=0$ and/or $\theta_j'(0)=v/2$. This implies that along any trajectory of (8),(9) starting from a point $z(0) = (A(0),A'(0),\theta(0),\theta'(0))$ in the set $Z \triangleq \{(A,A',\theta,\theta') \in \mathbb{R}^{4N}: A_j(v-2\theta_j') = 0,\ j=1,\ldots,N\}$, its corresponding phase $\theta(\xi) \triangleq (\theta_1(\xi),\ldots,\theta_N(\xi))$ has the form (19). In this case, f no longer depends on $A(0)$ and $\theta'(0)$, and (13) reduces to

$$d^2A_j/d\xi^2 = \partial U_1/\partial A_j, \quad j=1,\ldots,N. \tag{20}$$

The equilibrium points of (20) are points $(A_e,0)$ in $\mathbb{R}^{2N}$ such that A_e are the stationary points of U_1 or the roots of the equation $f(0,C,A)A = 0$. Obviously, A_e's include $A = 0$ and all those A's satisfying $g(\|A\|^2)=v^2(v^2-1)/4 - C$.

To obtain some qualitative information on the solutions of (20), we first derive a differential equation for $u(\xi) \triangleq \|A(\xi)\|^2$. By direct computation,

$$d^2u/d\xi^2 = 2\|A'(\xi)\|^2+2A(\xi)\cdot d^2A/d\xi^2 = 2\|A'(\xi)\|^2 + 2u\tilde{f}(C,u), \tag{21}$$

where $\tilde{f}(C,\|A\|^2) \triangleq f(0,C,A)$ as defined by (12). Along an integral curve (17) corresponding to a fixed C_1 and $\mu=0$, (21) can be rewritten as

$$d^2u/d\xi^2 = 2\{u\tilde{f}(C,u) + C_1 + 2U_1(u,C)\} \triangleq P(u,C,C_1). \tag{22}$$

Its solution starting with initial conditions

$$u(0) = \|A(0)\|^2, \quad u'(0) = 2A(0)\cdot A'(0) \tag{23}$$

satisfying

$$\|A'(0)\|^2 = C_1 + 2U_1(\|A(0)\|^2, C) \geq 0 \tag{24}$$

describes the evolution of $\|A(\xi)\|$ with ξ along the integral curve.

A first integral of (22) is given by

$$(u'(\xi))^2 = (u'(0))^2 + \int_{u(0)}^{u(\xi)} 2P(\eta, C, C_1)\,d\eta \triangleq Q(u, C, C_1, u'(0)), \tag{25}$$

where $u' = du/d\xi$. Equation (25) is valid only when its right-hand-side is nonnegative. An implicit expression for $\|A(\xi)\|^2$ can be obtained by integrating (25):

$$\int_{\|A(0)\|^2}^{\|A(\xi)\|^2} Q(\eta, C, C_1, u'(0))^{-\frac{1}{2}}\,d\eta = \pm\xi\ . \tag{26}$$

Note that if an explicit expression for $\|A(\xi)\|^2$ is obtainable from (26), then $A(\xi)$ can be determined by integrating each equation in *(11)* independently with $\mu=0$.

In the sequel, we shall establish conditions for the existence or nonexistence of solutions of (20) having the property that $\|A(\xi)\| \to 0$ as $|\xi| \to \infty$, or solitary-wave solutions of (7) with $\mu=0$.

<u>Theorem 1</u>: If

$$\kappa g(u) + \gamma \geq 0 \tag{27}$$

for all $u \geq 0$, then there do not exist solutions of *(20) such that* $\|A(0)\| > 0$ and $\|A(\xi)\| \to 0$ as $|\xi| \to \infty$.

<u>Proof</u>: Condition (27) is equivalent to $\tilde{f}(C,u) \geq 0$ for all $u \geq 0$. In view of (21), we have $d^2u/d\xi^2 \geq 0$ implying that $\|A(\xi)\|^2$ along any solution of (20) is a convex function of ξ. Hence, it is impossible to have $\|A(0)\| > 0$ and

$\|A(\xi)\| \to 0$ as $|\xi| \to \infty$. ∎

Note that for the subsonic ($v^2<1$) and supersonic ($v^2>1$) cases, (27) implies that $g(u)$ is uniformly bounded above and below by $v^2(v^2-1)/4-C$ respectively. Also, if (27) is a strict inequality, then $(A,A')=(0,0)$ is the only equilibrium point of (20).

Theorem 2: Assume that the following conditions are satisfied:

(i) $v^2(v^2-1) > 4C$ and $v^2<1$;

(ii) g is a strictly monotone increasing function in $C_1(\mathbb{R},\mathbb{R})$ with $g(0)=0$ and there exists a positive number $u_1<\infty$ such that

$$\int_0^{u_1} g(\eta)d\eta = \{v^2(v^2-1)/4-C\}u_1 \tag{28}$$

and

$$\int_0^{u} g(\eta)d\eta > \{v^2(v^2-1)/4-C\}u \quad \text{for all } u>u_1. \tag{29}$$

Then (20) has a solution $A(\xi)\geqslant 0$ for all $\xi\in\mathbb{R}$, with $\|A(0)\|>0$ and $\|A'(0)\|=0$ such that

$$\|A(\xi)\| \text{ and } \|A'(\xi)\| \to 0 \text{ as } |\xi| \to \infty. \tag{30}$$

Proof: First, we note from (15) and (17) with $\mu=0$ that for a solution to have property (30), the initial condition $(A(0),A'(0))$ must satisfy

$$\tilde{C}_1 \triangleq \|A'(0)\|^2 - 2U_1(\|A(0)\|^2,C) = 0. \tag{31}$$

We shall show that under condition (i), $\tilde{C}_1=0$ implies property (30). From (17), it is evident that when $(A(0),A'(0))$ satisfies (31), its corresponding trajectory is a zero-level curve of $I(A,A')$ defined by

$$I(A,A') \triangleq \|A'\|^2 - 2U_1(\|A\|^2,C) = 0, \tag{32}$$

or the points along the trajectory belong to the set

$$I^{-1}(0) = \{(A,A') \in \mathbb{R}^{2N} : \|A'\|^2 = 2U_1(\|A\|^2, C)\}. \tag{33}$$

Obviously, the equilibrium point $(A,A')=(0,0) \in I^{-1}(0)$. Now the foregoing implications can be established by verifying that (0,0) is the only equilibrium point in $I^{-1}(0)$, moreover, it is a saddle point.

Let $(A_e, 0)$ be an equilibrium point of (20) with $\|A_e\| > 0$. Then, A_e must satisfy $f(0,C,A_e)=0$ or

$$4g(\|A_e\|^2) = v^2(v^2-1) - 4C. \tag{34}$$

Suppose that $(A_e, 0) \in I^{-1}(0)$. Then, we must have

$$2U_1(\|A_e\|^2, C) \triangleq \int_0^{\|A_e\|^2} \{\kappa g(\eta) + \gamma\} d\eta = 0. \tag{35}$$

From (34), we have

$$2U_1(\|A_e\|^2, C) = \int_0^{\|A_e\|^2} \kappa\{g(\eta) - g(\|A_e\|^2)\} d\eta \tag{36}$$

which is a positive quantity for $\|A_e\| > 0$ under condition (ii). This contradicts (35). Hence $(A,A')=(0,0)$ is the only equilibrium point in $I^{-1}(0)$.

To show that $(A,A')=(0,0)$ is a saddle point, consider the following linearized equation (20) about $(A,A')=(0,0)$:

$$d^2\delta A_j/d\xi^2 = (\partial^2 U_1/\partial A_j^2)|_{A=0}\delta A_j, \quad j=1,\ldots,N, \tag{37}$$

where

$$(\partial^2 U_1/\partial A_j^2)|_{A=0} = \gamma \tag{38}$$

Note that due to the symmetry of U_1 about $A=0$, $(\partial^2 U_1/\partial A_j \partial A_k)|_{A=0} = 0$ for $j \neq k$.

Under condition (i), $(\partial^2U_1/\partial A_j^2)|_{A=0}>0$, $j=1,\ldots,N$, so $(A,A')=(0,0)$ is a saddle point. Moreover, it is the limit point of some trajectory lying in $I^{-1}(0)$ as $|\xi|\to\infty$. Hence, $C_1=0$ implies property (30).

Next, we must verify that there exist points $(A(0),A'(0))=(A(0),0)$ with $\|A(0)\|>0$ such that $C_1=0$. From (15) and (31), this corresponds to finding a $\|A(0)\|>0$ such that

$$2U_1(\|A(0)\|^2,C) \stackrel{\Delta}{=} \int_0^{\|A(0)\|^2} \{\kappa g(\eta)+\gamma\}\,d\eta = 0, \tag{39}$$

which inview of (29), can be rewritten as

$$\|A(0)\|^2 = W(\|A(0)\|^2) \stackrel{\Delta}{=} 4\{v^2(v^2-1)-4C\}\int_0^{\|A(0)\|^2} g(\eta)d\eta. \tag{40}$$

Evidently, under condition (ii), the mapping W has a nonzero fixed point $\|A(0)\|^2<\infty$.

We have established that there exist points $(A(0),0)$ in $I^{-1}(0)$ with $\|A(0)\|>0$. Now, we must show that for such a point, there exists a trajectory curve lying in $I^{-1}(0)$ which joins $(A(0),0)$ and $(0,0)$. This is assured when $I^{-1}(0)$ is compact. It is straightforward to show that $I^{-1}(0)$ is closed. To show that $I^{-1}(0)$ is bounded, we rewrite (32) as

$$w = \int_0^u \{\kappa g(\eta) + \gamma\}d\eta, \tag{41}$$

where $w=\|A'\|^2$ and $u=\|A\|^2$. Condition (i) implies that $\gamma>0$ and $\kappa<0$. From (ii), there exists a finite $u_1>0$ such that the right-hand-side of (41) is zero at u_1 and negative for all $u > u_1$. Since (41) is valid only for $w\geq0$, hence $\|A\|^2\leq u_1$. Also, from Weierstrass theorem, there exists a finite $W_1>0$ such that $\|A'\|^2\leq W_1$, since the right-hand-side of (41) is continuous on

the compact interval $0 \leq u \leq u_1$. Thus, the boundedness of $I^{-1}(0)$ is established.

Finally, since only the nonnegative solutions of (20) are meaningful here, it remains to show that for a point $(A(0),0) \in I^{-1}(0)$ with $A(0) \geq 0$ and $\|A(0)\| > 0$, its corresponding solution is nonnegative, that is, $A(\xi) \geq 0$ for all $\xi \in \mathbb{R}$. This is immediately apparent from the fact that $I^{-1}(0) = I_+^{-1}(0) \cup I_-^{-1}(0)$ and $I_+^{-1}(0) \cap I_-^{-1}(0) = \{(0,0)\}$, where $I_+^{-1}(0) = \{(A,A') \in I^{-1}(0): A \geq 0\}$ and $I_-^{-1}(0) = \{(A,A') \in I^{-1}(0): A \leq 0\}$, since $U_1(0,C) = 0$. ∎

Remarks: (R-1) Theorems 1 and 2 give respectively sufficient conditions for the nonexistence and existence of multidimensional solitary-wave solutions of (1) which are directly analogous to those for the one-dimensional case. From (33), it is evident that $I^{-1}(0)$ is symmetric about $A=0$ and $A'=0$. Also, $I_+^{-1}(0)$ and $I_-^{-1}(0)$ are symmetric about $A'=0$. Thus, under the conditions of Theorem 2, the trajectory curves in the (A,A')-space corresponding to the solitary-wave solutions of (7) satisfying (30) have similar properties, and they have the form:

$$h_j(\xi) = A_j(\xi)\exp\{i(\theta_j(0)+v\xi/2)\}, \quad j=1,\ldots,N. \tag{42}$$

(R-2) Along any solution of (20), its corresponding density $s(\xi)$ can be found directly by solving (26) for $\|A(\xi)\|^2$ or $|h(\xi)|^2$ and substituting the result into (6) with $\hat{C}=0$. Complete knowledge of the solution $A(\xi)$ is not necessary here.

We note that if the assumptions in Theorem 2 are satisfied and there exists a positive number r_e such that

$$\kappa g(r_e^2) + \gamma = 0 \quad \text{or} \quad g(r_e^2) = v^2(v^2-1)/4 - C, \tag{43}$$

then (20) has an uncountably infinite number of nonisolated, nonzero equilibrium points $(A_e,0)$ such that A_e lies on the sphere $\{A_e \in \mathbb{R}^N: \|A_e\| = r_e\}$.

Now, we show that there exist solutions $A(\xi)$ of (20) in some neighborhood of these equilibrium points such that their norms are periodic functions of ξ.

First, we rewrite (22) in the form

$$d^2u/d\xi^2 = \partial V(u,C_1)/\partial u, \tag{44}$$

where

$$V(u,C_1) = 2\int_0^u \{\eta(\kappa g(\eta) + \gamma) + C_1 + 2U_1(\eta,C)\}d\eta, \tag{45}$$

and the initial conditions $u(0)=\|A(0)\|^2$ and $u'(0)=2A(0)\cdot A'(0)$ are chosen such that condition (24) is satisfied. It can be readily verified that if we set $C_1=C_1^o$ given by

$$C_1^o = -\int_0^{r_e^2} \{\kappa g(\eta) + \gamma\}d\eta, \tag{46}$$

then $u_e=r_e^2$ is a stationary point of $V(\cdot,C_1^o)$, or $(u,u')=(r_e^2,0)$ is an equilibrium point of (44). For $C_1=C_1^o$, condition (24) becomes

$$\|A'(0)\|^2 = \int_{r_e^2}^{\|A(0)\|^2} \{\kappa g(\eta) + \gamma\}d\eta \geq 0. \tag{47}$$

Under condition (i) and (ii) of Theorem 2, we have $\kappa g(\eta)+\gamma>0$ for $0\leq\eta<r_e^2$, and $\kappa g(\eta)+\gamma<0$ for all $\eta>r_e^2$. Evidently, (47) is satisfied if and only if $\|A(0)\| = r_e$. Hence, the only solution to (44) with initial condition $(u(0),u'(0))$ satisfying (47) is the equilibrium solution $(u(\xi),u'(\xi))=(r_e^2,0)$ for all ξ.

Now, we consider the solutions of (44) with $C_1=C_1^o+\delta C_1$ and initial condition $(u(0),u'(0))=(\|A(0)\|^2,2A(0)\cdot A'(0))$ satisfying (24), where δC_1 is a small perturbation of C_1 about C_1^o. Let $u_e(C_1)$ denote a stationary point

of $V(\cdot,C_1)$ or a root of the equation

$$u\{\kappa g(u) + \gamma\} + C_1 + 2U_1(u,C) = 0. \tag{48}$$

For $C_1 = C_1^o + \delta C_1$, we can write

$$u_e(C_1) = r_e^2 + \delta u_e. \tag{49}$$

Clearly, under the assumptions of Theorem 2, δu_e depends continuously on δC_1 and $|\delta u_e| \to 0$ as $|\delta C_1| \to 0$. Also, since

$$\partial^2 V(u,C_1)/\partial u^2 = 4\{\kappa g(u) + \gamma\} + 2\kappa u g'(u), \tag{50}$$

we have $(\partial^2 V(u,C_1)/\partial u^2)|_{u=r_e} = 2\kappa r_e g'(r_e) < 0$, or $u_e = r_e^2$ is a relative maximum point of $V(\cdot,C_1^o)$. In fact, since $2\kappa u g'(u) < 0$ for all $u > 0$ and $\kappa g(r_e^2) + \gamma = 0$ there exists a positive number ε such that for each δC_1, $|\delta C_1| < \varepsilon$, its corresponding $u_e(C_1) = r_e^2 + \delta u_e$ is a relative maximum point of $V(\cdot, C_1^o + \delta C_1)$. Consequently, for any fixed δC_1, $|\delta C_1| < \varepsilon$, (44) has periodic solutions in some neighborhood of the corresponding equilibrium point $(u,u') = (r_e^2 + \delta u_e, 0)$.[11] They are given by the solution of

$$(u'(\xi))^2/2 = V(u(\xi), C_1 + \delta C_1) - V(u_o, C_1^o + \delta C_1) + (u_o')^2/2 \tag{51}$$

with $u(0) = u_o$, where the initial point $(u(0), u'(0)) = (u_o, u_o')$ is sufficiently close to $(u,u') = (r_e^2 + \delta u_e, 0)$. In particular, we can choose $u_o = \|A(0)\|^2 > 0$ and $u_o' = 2A(0)\cdot A'(0)$ such that condition (24) given by

$$\begin{aligned} \|A'(0)\|^2 &= C_1^o + \delta C_1 + 2U_1(\|A(0)\|^2; C) \\ &= \delta C_1 + \int_{r_e^2}^{\|A(0)\|^2} \{\kappa g(\eta) + \gamma\} d\eta \geq 0 \end{aligned} \tag{52}$$

is satisfied. This is possible for any positive δC_1. The existence of

solutions of (20) in some neighborhood of the equilibrium points $(A_e, 0)$ with $\|A_e\| = r_e$, whose norms are periodic functions of ξ follows from the fact that $|\delta u_e| \to 0$ as $|\delta C_1| \to 0$. The foregoing result can be summarized as a theorem.

Theorem 3: Assume that the conditions of Theorem 2 are satisfied, and there exists a real number $r_e > 0$ satisfying (43). Then there exist solutions $A(\xi)$ of (20) in some neighborhood of the equilibrium set $\{(A, A') \in \mathbb{R}^{2N}: \|A\| = r_e,\ A' = 0\}$ such that their norms $\|A(\xi)\|$ are periodic functions of ξ.

Note that in the multidimensional case, the periodicity of $u(\xi) = \|A(\xi)\|^2$ generally does not imply the periodicity of $A(\xi)$. Since the energy density of the electric field is proportional to $|h(\xi)|^2$, solutions with periodic $|h(\xi)|$ represent oscillatory energy densities. Evidently, from (6) (with $\hat{C} = 0$), the periodicity of $s(\xi)$ is implied by that of $|h(\xi)|$. Now, we give a simple sufficient condition for the nonexistence of periodic travelling waves in the sense that $|h(\xi)|$ and $s(\xi)$ are periodic in ξ.

Theorem 4: Suppose that the following conditions are satisfied:

(i) g is a real-valued continuous monotone increasing function defined on $\mathbb{R}$ such that $g(0) = 0$;

(ii) $v^2(v^2 - 1) < 4C$ and $v^2 > 1$;

(iii) the initial conditions $A(0)$ and $A'(0)$ satisfy $\|A(0)\| > 0$ and $\tilde{C}_1 \geq 0$, where $\tilde{C}_1$ is defined in (31).

Then, the norm of the corresponding solution $A(\xi)$ of (20) is nonperiodic in ξ.

Proof: Consider (22) given explicitly by:

$$d^2u/d\xi^2 = 4\gamma u + 2\tilde{C}_1 + 2\kappa \left\{ ug(u) + \int_0^u g(\eta)\, d\eta \right\}, \tag{53}$$

where γ and κ are as in (16). Under condition (i), the $\{\ldots\}$ term in (53) is nonnegative for $u \geq 0$. From conditions (ii) and (iii), we have $\gamma > 0$ so that $d^2u/d\xi^2 \geq 0$ for all $u \geq 0$. Since $u(0) = \|A(0)\|^2 > 0$, u is a nonzero convex function of ξ which cannot be periodic. ∎

Theorem 4 gives a sufficient condition for the nonexistence of supersonic periodic travelling waves. In the subsonic case ($v^2 < 1$), the condition $v^2(v^2-1) < 4C$ implies that $\gamma < 0$. Thus, under condition (i) of Theorem 4, we have $d^2u/d\xi^2 \leq 0$ for all $u \geq 0$ when $\tilde{C}_1 \leq 0$, which implies the nonexistence of subsonic periodic travelling waves. However, $\tilde{C}_1 \leq 0$ corresponds to

$$\|A'(0)\|^2 \leq \gamma\|A(0)\|^2 + \kappa\int_0^{\|A(0)\|^2} g(\eta)d\eta, \tag{54}$$

whose right-hand-side is nonpositive. Thus, this condition can be satisfied only in the trivial case when $A(0)=0$ and $A'(0)=0$.

3. SPECIAL CASES

Now, we apply the results in Section 2 to equation (1) with particular forms of g arising in physical situations.

3.1 $g(|E|^2) = |E|^2$: This corresponds to the case with the usual pondermotive force. Here, U_1 as given by (15) has the explicit form:

$$2U_1(\|A\|^2, C) = \gamma\|A\|^2 + \kappa\|A\|^4/2, \tag{55}$$

where γ and κ are as in (16). A first integral of (13) is given by

$$\|A'(\xi)\|^2 - \gamma\|A(\xi)\|^2 - \kappa\|A(\xi)\|^4/2 - \sum_{j=1}^{N} \mu_j^2 A_j^{-2}(\xi)/4 = C_1. \tag{56}$$

When $A(0)$ and $\theta(0)$ are chosen such that $\mu=0$, the equation for $A_j(\xi)$ given by (20) reduces to

$$d^2A_j/d\xi^2 = (\gamma + \kappa\|A\|^2)A_j, \quad j=1,\ldots,N, \tag{57}$$

and the equation for $u(\xi)=\|A(\xi)\|^2$ given by (22) becomes

$$d^2u/d\xi^2 = 3\kappa u^2 + 4\gamma u + 2C_1. \tag{58}$$

A first integral of (53) is given by

$$(u'(\xi))^2/2 = \kappa u^3(\xi) + 2\gamma u^2(\xi) + 2C_1 u(\xi) + C_2, \tag{59}$$

where C_2 is an integration constant. By restricting the right-hand-side of (59) to be nonnegative, we have the following implicit expression for $u(\xi)$:

$$\int_{u(0)}^{u(\xi)} \{\kappa\eta^3 + 2\gamma\eta^2 + 2C_1\eta + C_2\}^{-\frac{1}{2}} d\eta = \sqrt{2}\xi, \qquad \xi \in \mathbb{R}. \tag{60}$$

Now, we apply Theorem 1 to this special case. Clearly, for $v^2<1$, condition (27) cannot be satisfied. But for $v^2>1$, (27) is satisfied when $v^2(v^2-1)/4=C$. Under this condition, there do not exist solutions of (20) or solitary-wave solutions of (7) such that $|h(0)|>0$ and $|h(\xi)|\to 0$ as $|\xi|\to\infty$. *To apply Theorem 2 to this special case, we see that* under condition (i), $g(u)=u$ satisfies condition (ii). Thus, under condition (i), (57) has solitary-wave solutions satisfying (30). For such solutions, we see from (56) with $\mu=0$ that C_1 must be zero. Also, from (59), C_2 must be zero, since $u(0)=\|A(0)\|^2>0$ and $u'(0)=0$, $\|A(0)\|$ must satisfy

$$\|A(0)\|^2 = -2\gamma/\kappa, \tag{61}$$

where $\gamma>0$ and $\kappa<0$ under condition (i). Thus, (60) becomes

$$\int_{2\gamma/|\kappa|}^{\|A(\xi)\|^2} \{\kappa\eta^3 + 2\gamma\eta^2\}^{-\frac{1}{2}} d\eta = \sqrt{2}\xi, \qquad \xi \in \mathbb{R}. \tag{62}$$

It follows that

$$\|A(\xi)\| = (2\gamma/|\kappa|)^{\frac{1}{2}} \operatorname{sech}(\sqrt{\gamma}\xi), \quad \xi \in \mathbb{R}, \tag{63}$$

and in view of (6), we have

$$s(\xi) = (v^2-1)^{-1}\{(2\gamma/|\kappa|) \operatorname{sech}^2(\sqrt{\gamma}\xi) + C\}. \tag{64}$$

Substituting (64) into (11) with $\mu_j=0$ leads to a set of uncoupled equations for A_j's given by

$$d^2A_j/d\xi^2 = (\gamma + \kappa\|A\|^2)A_j = \gamma\{1 - 2\operatorname{sech}^2(\sqrt{\gamma}\xi)\}A_j, \quad j=1,\dots,N, \tag{65}$$

which can be integrated independently to obtain $A_j(\xi)$. The foregoing results are consistent with those for the one-dimensional solitary waves.[10]

Turning now to the periodic travelling-waves, we observe that under condition (i) of Theorem 2, (57) has an uncountably infinite number of nonisolated equilibrium points $(A_e,0)$ such that $\|A_e\|^2=r_e^2=-\gamma/\kappa>0$. Also, (58) can be written in the form of (44) with

$$V(u,C_1) = u(\kappa u^2 + 2\gamma u + 2C_1). \tag{66}$$

If we set $C_1=C_1^0 \triangleq \gamma^2/(2\kappa)$, then $u=-\gamma/\kappa$ is a relative maximum point of $V(\cdot,C_1^0)$. Now, we consider the solutions of (58) with initial conditions $u(0)=\|A(0)\|^2$ and $u'(0)=2A(0)\cdot A'(0)$ satisfying condition (24) given explicitly by

$$\|A'(0)\|^2 = C_1 + \gamma u(0) + \kappa u^2(0)/2 \geqslant 0. \tag{67}$$

Let $\mathcal{U}(C_1)$ denote the set of all $u(0)\geqslant 0$ satisfying (67) for a fixed C_1. It can be readily verified that under condition (i) of Theorem 2, we have

$$U(C_1)=\{u(0):[\gamma-(\gamma^2+2|\kappa|C_1)^{\frac{1}{2}}]/|\kappa|\leqslant u(0)\leqslant[\gamma+(\gamma^2+2|\kappa|C_1)^{\frac{1}{2}}]/|\kappa|\}$$

$$\text{for } 0\geqslant C_1\geqslant\gamma^2/(2\kappa) \qquad (68)$$

$$U(C_1)=\{u(0):0\leqslant u(0)\leqslant[\gamma+(\gamma^2+2|\kappa|C_1)^{\frac{1}{2}}]/|\kappa|\} \quad \text{for } C_1\geqslant 0, \qquad (69)$$

and $U(C_1)$ is empty for $C_1<\gamma^2/(2\kappa)$. Note that for $C_1=\gamma^2/(2\kappa)$, $U(C_1)$ contains only the point $u(0)=-\gamma/\kappa$. Thus, from Theorem 3, if condition (i) of Theorem 2 are satisfied, then there exist solutions $A(\xi)$ of (20) in some neighborhood of the equilibrium set $\{(A,A')\in\mathbb{R}^{2N}:\|A\|^2=-\gamma/\kappa, A'=0\}$ such that their norms $\|A(\xi)\|$ are periodic in ξ. These solution curves correspond to (56) with $\mu=0$ and C_1 satisfying $0>C_1>\gamma^2/(2\kappa)$. When C_1 is set to zero, we have solitary-wave solutions such that $\|A(\xi)\|$ and $\|A'(\xi)\|\to 0$ as $|\xi|\to\infty$ as given by (63). In this case, $(u,u')=(0,0)$ is a saddle point of (58) with $C_1=0$.

Fig.1 shows the trajectories of (58) with $\gamma=1$ and $\kappa=-2$ in the (u,u')-plane for various values of C_1 and $u(0)$ satisfying (67). Note that for $C_1>0$, $(u,u')=(0,0)$ is not an equilibrium point of (58). In fact, these solutions pass through the origin and they are periodic functions of ξ. Fig.2 shows the behavior of the trajectories in the ξ-domain.

Finally, for the supersonic case $v^2>1$, we have from Theorem 4 that if $v^2(v^2-1)<4C$ and $A(0),A'(0)$ satisfy $\|A(0)\|>0$ and $\tilde{C}_1\geqslant 0$ or

$$\|A'(0)\|^2 \geqslant \gamma\|A(0)\|^2+\kappa\|A(0)\|^4/2>0, \qquad (70)$$

then the norm of the corresponding solution $A(\xi)$ of (57) is nonperiodic in ξ.

3.2 $g(|E|^2)=K(1-\exp(-|E|^2))$: This form of g, with K being a positive constant, has been proposed by Wilcox and Wilcox[12] to represent ion density saturation. For this g, U_1 is given by

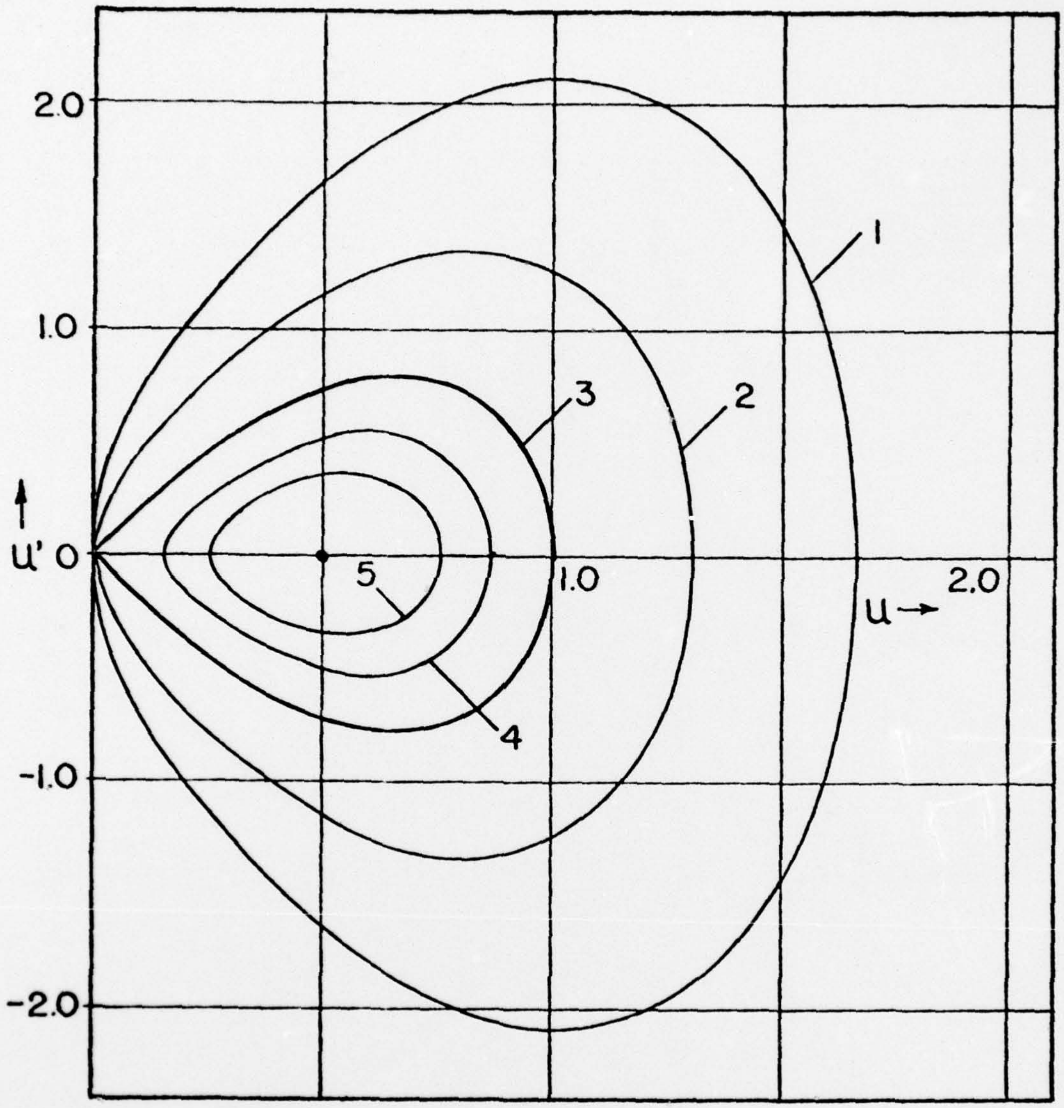

Fig.1: Trajectories of (58) with $\gamma=1$ and $\kappa=-2$ in the (u,u')-plane for $u(0)$ satisfying (76) and fixed values of C_1 (curves 1-5 correspond to $C_1=1.1, 0.4, 0.0, -1/8, -3/16$ respectively); curve 3 is the solitary-wave solution.

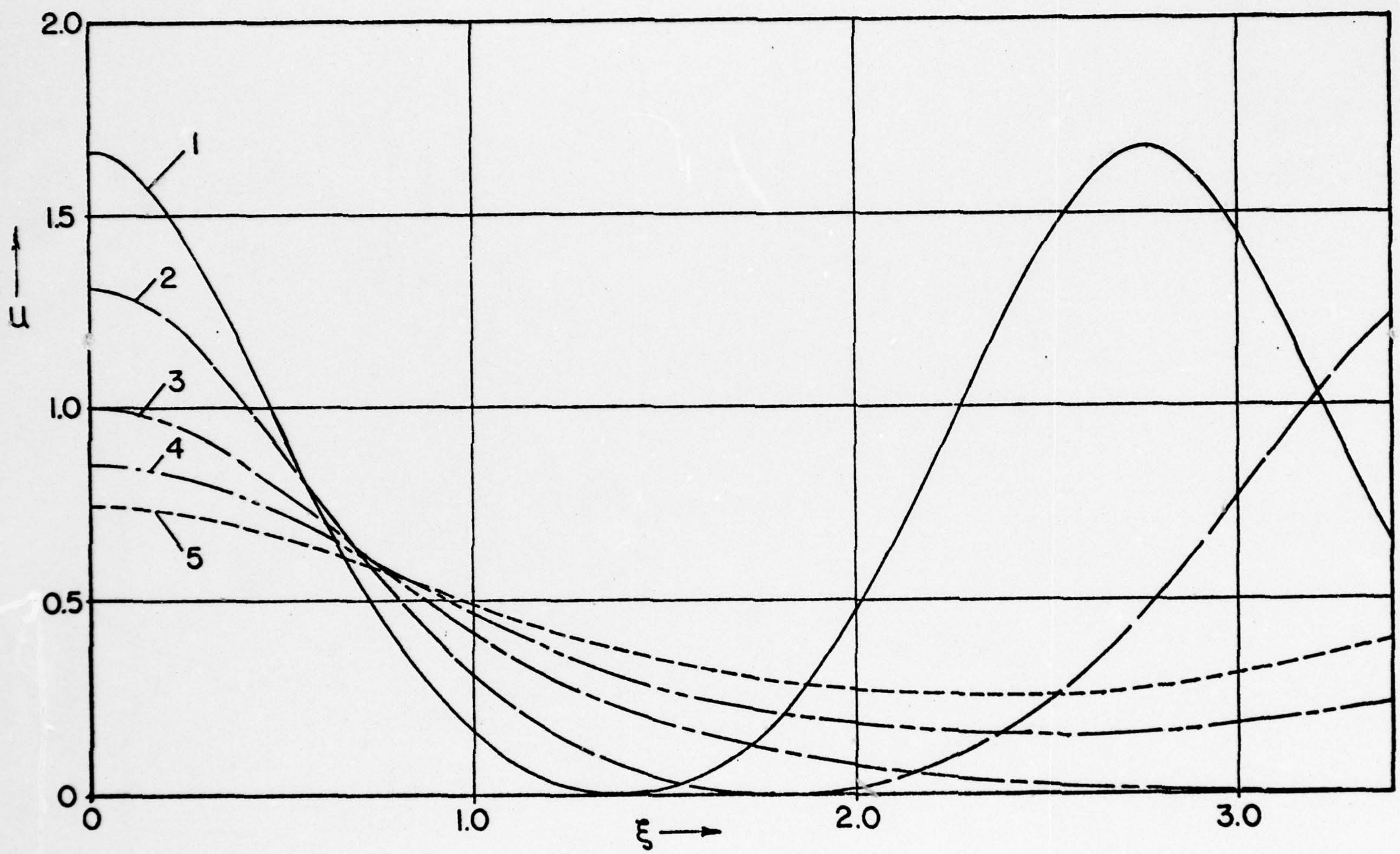

Fig.2: Behavior of the trajectories shown in Fig.1 in the ξ-domain.

$$2U_1(\|A\|^2,C) = (\kappa K+\gamma)\|A\|^2 - \kappa K\{1-\exp(-\|A\|^2)\}, \tag{71}$$

and a first integral of (13) is given by

$$\|A'(\xi)\|^2 - (\kappa K+\gamma)\|A(\xi)\|^2 + \kappa K\{1-\exp(-\|A(\xi)\|^2)\} - \sum_{j=1}^{N} \mu_j^2 A_j^{\,2}(\xi)/4 = C_1. \tag{72}$$

The equations for the $A_j(\xi)$'s with $\mu=0$ corresponding to (20) have the form:

$$d^2A_j/d\xi^2 = \{\gamma + \kappa K[1-\exp(-\|A(\xi)\|^2)]\}A_j, \quad j=1,\dots,N. \tag{73}$$

The evolution of $u(\xi) = \|A(\xi)\|^2$ with ξ along an integral curve of (73) specified by C_1 is governed by

$$d^2u/d\xi^2 = 2\{2(\gamma+\kappa K)u + C_1 - \kappa K[1-(1-u)\exp(-u)]\}, \tag{74}$$

which has a first integral of the form:

$$(u'(\xi))^2/2 = 2(\gamma+\kappa K)u^2(\xi) + 2u(\xi)[C_1 + \kappa K(\exp(-u(\xi))-1)] + C_2, \tag{75}$$

where C_2 is an integration constant. By restricting the right-hand-side of (75) to be nonnegative, we can integrate (75) to give an implicit expression for $u(\xi)$.

To apply Theorem 1 to this case, consider inequality (27) given explicitly by

$$\kappa K(1-\exp(-u)) + \gamma > 0 \qquad \text{for all } u \geq 0. \tag{76}$$

This condition is satisfied when

$$v^2 > 1 \quad \text{and } \gamma > 0 \tag{77}$$

or

$$v^2 < 1 \quad \text{and } \kappa K + \gamma > 0. \tag{78}$$

Thus, under (77) or (78), there do not exist solitary-wave solutions such

that $|h(0)| > 0$ and $|h(\xi)| \to 0$ as $|\xi| \to \infty$. Moreover, from Theorem 4, when $v^2 > 1$, $\gamma > 0$ and $(A(0), A'(0))$ satisfies

$$\|A'(0)\|^2 \geq (\kappa K+\gamma)\|A(0)\|^2 - \kappa K\{1-\exp(-\|A(0)\|^2)\}, \tag{79}$$

then the norm of the corresponding solution $A(\xi)$ of (73) is nonperiodic in in ξ.

Now, consider condition (29) in Theorem 2 which requires the existence of a $u_1 > 0$ such that

$$K[u_1 + \exp(-u_1)-1] = (-\gamma/\kappa)u_1 \tag{80}$$

and for all $u > u_1$,

$$K[u + \exp(-u)-1] > (-\gamma/\kappa)u \tag{81}$$

This condition is satisfied if

$$\gamma/\kappa < 0. \tag{82}$$

Thus, if $v^2 < 1$ and $\gamma > 0$, then the hypotheses of Theorem 2 are satisfied. Hence (73) has solitary-wave solutions satisfying (30). *For such a solution* with $\|A'(0)\| = 0$, we have from (72) with $C_1 = 0$ and $\mu = 0$ that $\|A(0)\|$ must satisfy

$$\|A(0)\|^2 = \kappa K(\kappa K+\gamma)^{-1}\{1-\exp(-\|A(0)\|^2)\}, \tag{83}$$

which always has a solution $\|A(0)\|^2 > 0$ if $v^2 < 1$ and $\gamma > 0$.

Next, we observe that if

$$\kappa K/(\gamma+\kappa K) > 1, \tag{84}$$

then (73) has an uncountably infinite number of nonisolated equilibrium points $(A, A') = (A_e, 0)$ such that

$$u_e \overset{\Delta}{=} \|A_e\|^2 = \ln[\kappa K/(\gamma+\kappa K)] > 0. \tag{85}$$

Note that if $v^2<1$, then (84) implies (82) and

$$(\gamma + \kappa K) < 0. \tag{86}$$

We shall verify that under the conditions of Theorem 2, (74) has periodic solutions in some neighborhood of the point $(u,u')=(u_e,0)$.

First, we rewrite (74) in the form of (44) with V given by

$$V(C_1,u) = 2(\gamma+\kappa K)u^2 + 2(C_1-\kappa K)u + 2\kappa Ku \exp(-u). \tag{87}$$

If we set $C_1=C_1^o$ given by

$$C_1^o = 2(\gamma+\kappa K)\ln[(\gamma+\kappa K)/(\kappa K)] + \kappa K - (\gamma+\kappa K)[1-\ln(\kappa K/(\gamma+\kappa K))], \tag{88}$$

then u_e given by (85) is a stationary point of $V(\cdot,C_1^o)$, or $(u,u')=(u_e,0)$ is an equilibrium point of (74). At this point, $(\partial^2V(u,C_1^o)/\partial u^2)|_{u=u_e} = 2u_e(\gamma+\kappa K)$. Thus, under condition (86), u_e is a relative maximum point of $V(\cdot,C_1^o)$. Now, we consider the solutions of (74) for various values of C_1 in some neighborhood of C_1^o, with initial conditions $u(0)=\|A(0)\|^2$ and $u'(0)=2A(0)\cdot A'(0)$ satisfying condition (24) given by

$$\|A'(0)\|^2 = C_1 + (\gamma+\kappa K)u(0) - \kappa K[1-\exp(-u(0))] \geqslant 0. \tag{89}$$

As in Section 3.1, let $\mathcal{U}(C_1)$ denote the set of all $u(0)\geqslant 0$ satisfying (89) for a fixed C_1 or

$$\mathcal{U}(C_1) = \{u(0)\geqslant 0: C_1+(\gamma+\kappa K)u(0) \geqslant \kappa K[1-\exp(-u(0))]\}. \tag{90}$$

It can be readily verified that if $v^2<1$ and condition (84) is satisfied, then $\mathcal{U}(C_1)$ is empty for all $C_1 < C_1^*$, and

$$\mathcal{U}(C_1^*) = \{u^*(0)\}, \tag{91}$$

where

$$u^*(0) = \ln[\kappa K/(\gamma+\kappa K)], \tag{92}$$

$$C_1^* = (\gamma+\kappa K)\ln[\kappa K/(\gamma+\kappa K)], \tag{93}$$

where $u^*(0)$ corresponds to the point of tangency between the line $y_1(u)= C_1^*(\kappa K)^{-1}+ [1+\gamma(\kappa K)^{-1}]u$ and the curve $y_2(u)=1 - \exp(-u)$. Also,

$$\mathcal{U}(C_1) = \{u(0): \check{u} \leqslant u(0) \leqslant \hat{u}\} \quad \text{for } C_1^* \leqslant C_1 \leqslant 0, \tag{94}$$

$$\mathcal{U}(C_1) = \{u(0): 0 \leqslant u(0) \leqslant \hat{u}\} \quad \text{for } C_1>0, \tag{95}$$

where $\check{u}$ and $\hat{u}$ with $\check{u}<\hat{u}$ are the two distinct positive roots of the equation

$$C_1 + (\gamma+\kappa K)u - \kappa K[1-\exp(-u)] = 0. \tag{96}$$

Thus, from Theorem 3, if $v^2<1$ and (84) is satisfied, then there exist solutions $A(\xi)$ of (73) in some neighborhood of the equilibrium set $\{(A,A')\in\mathbb{R}^{2n}: \|A\|^2= \ln[\kappa K/(\gamma+\kappa K)],\ A'=0\}$ such that their norms $\|A(\xi)\|$ are periodic in ξ. These solution curves correspond to (72) with $\mu=0$ and C_1 satisfying $0>C_1>C_1^*$. The trajectories of (74) with $\gamma=1,\kappa=-2$ and $K=1$ for various values of C_1 and $u(0)$ satisfying (89) are shown in Fig.3. Their corresponding trajectories in the ξ-domain are shown in Fig.4.

4. CONCLUSION

We have shown that under mild conditions on the nonlinearity g, (1) has multidimensional solitary-wave and periodic travelling-wave solutions $(E(\xi),n(\xi))$ in the sense that $|E(\xi)|$ and $n(\xi)$ tend to finite values as $|\xi| \to\infty$, and they are periodic functions of ξ respectively. Along these solutions, the phase of $E(\xi)$ is an affine function of ξ. Moreover, $u(\xi)=|E(\xi)|^2$

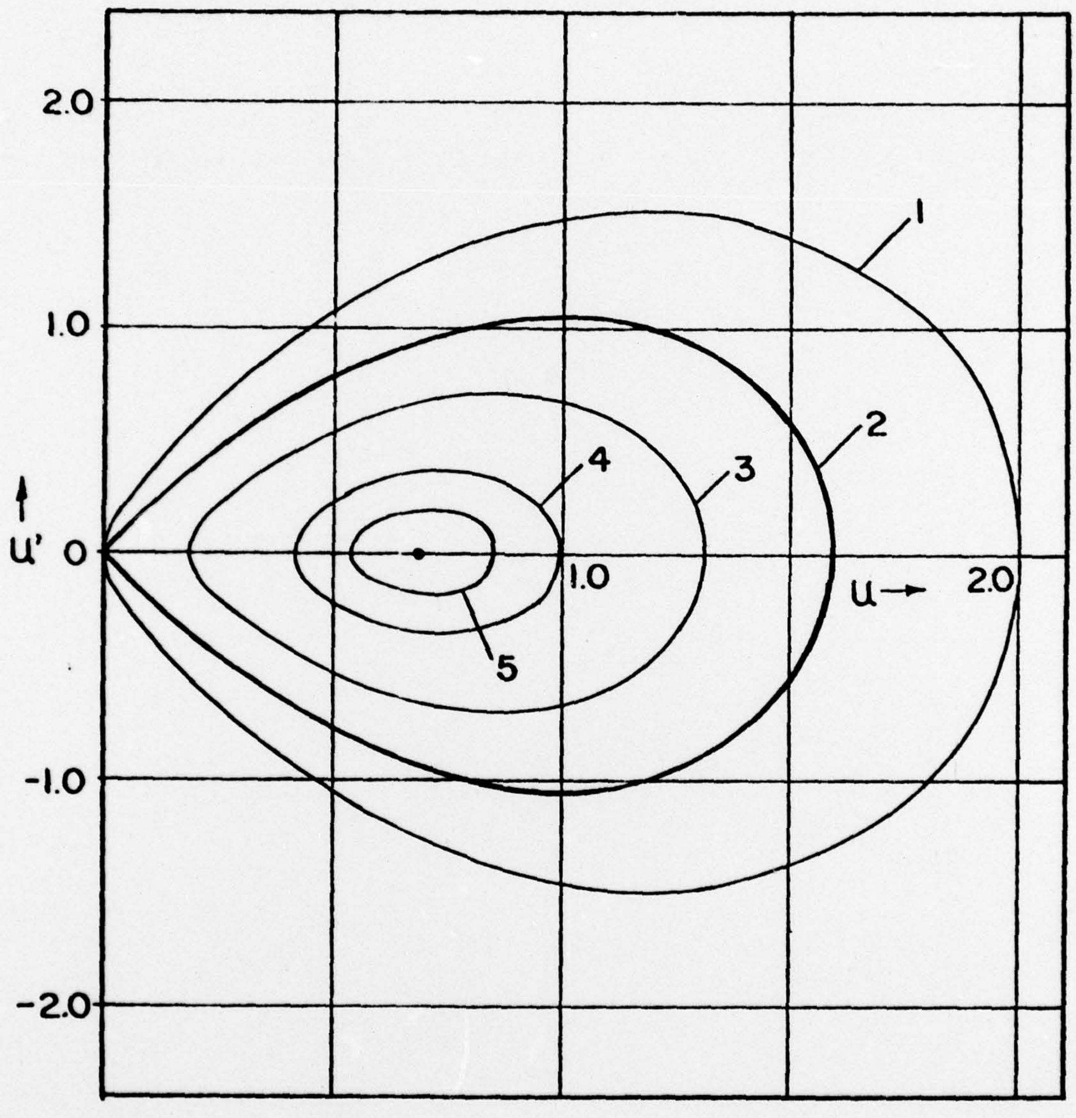

Fig. 3: Trajectories of (74) with $\gamma=1$ and $\kappa=-2$ in the (u,u')-plane for $u(0)$ satisfying (89) and fixed values of C_1 (curves 1-5 correspond to $C_1=0.2707$, 0.0, -0.1522, -0.2642, -0.3069 respectively); curve 2 is the solitary-wave solution.

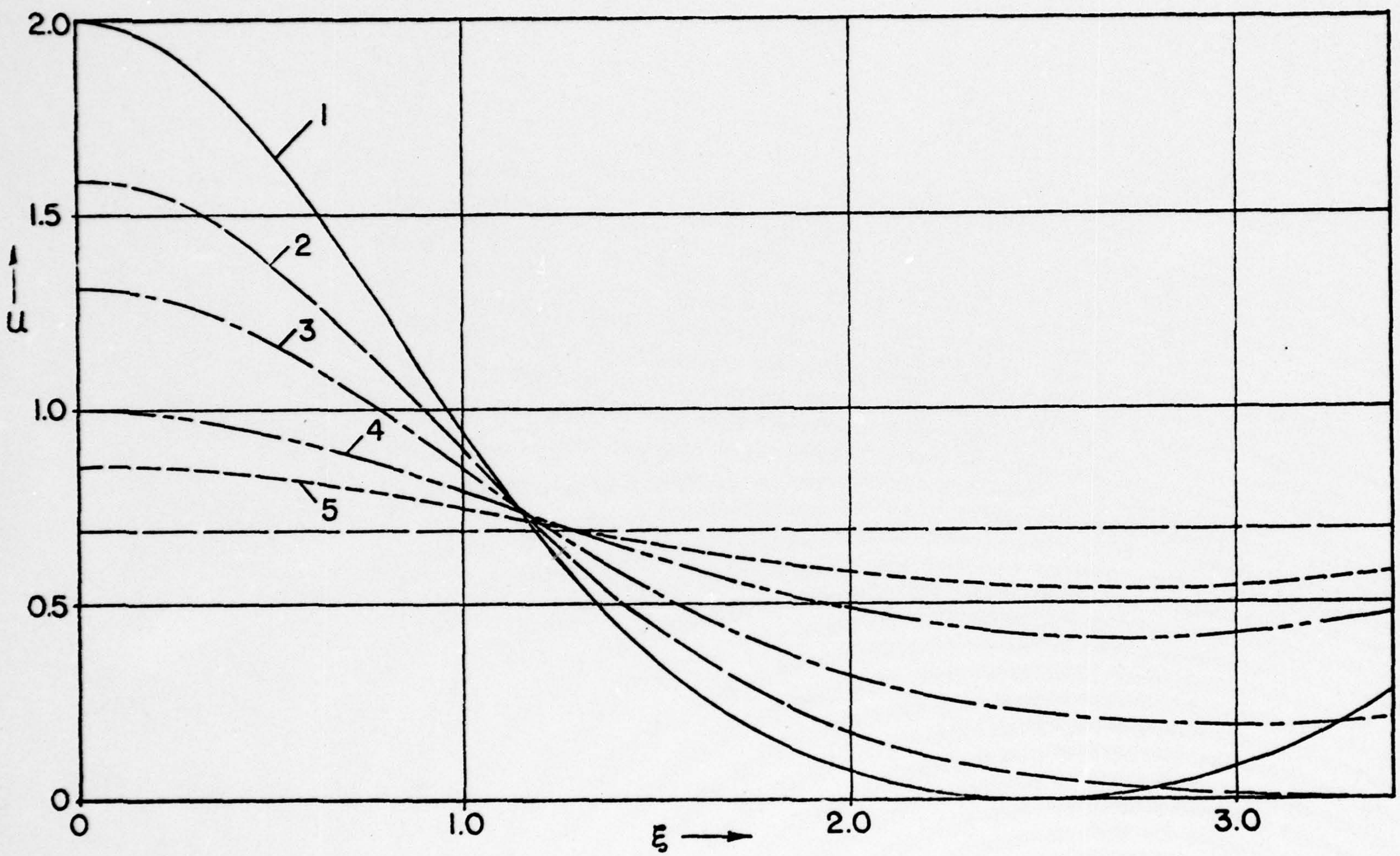

Fig.4: Behavior of the trajectories shown in Fig.3 in the ξ-domain.

satisfies a scalar second-order ordinary differential equation whose solutions have properties similar to those in the one-dimensional case. Although in this study, we have treated only the case with electrostatic waves (i.e. $\nabla\times E = 0$), the same approach may be used to obtain results for electromagnetic waves.

REFERENCES

1. V.E.Zakharov, Sov. Phys. JETP, 35,908 (1972).
2. V.D.Shapiro, Sov. Phys. JETP, 39,459 (1974).
3. L.M.Degtyarev, V.G.Nakhan'kov and L.I.Rudakov, Sov. Phys. JETP, 40,264 (1975).
4. L.M.Degtyarev, V.E.Zakharov and L.I.Rudakov, Sov. Phys. JETP, 41,57 (1975).
5. V.E.Zakharov, A.F.Mastryukov and V.S.Synakh, Sov. J. Plasma Phys. 1,339 (1975).
6. Kh.O.Abdulloev, I.L.Bogolyubskij and V.G.Makhan'kov, Nuclear Fusion, 15, 21 (1975).
7. V.V.Gorev, A.S.Kingsep and V.V.Yan'kov, Sov. Phys. JETP, 43,479 (1976).
8. V.N.Tsytovich, in Plasma Physics, Nonlinear Theory and Experiments, edited by H.Wilhelmsson (Plenum Press, New York,1977), p.166.
9. K.Kato, K.Mima, M.Watanabe and K.Nishikawa, in Plasma Physics, Nonlinear Theory and Experiments, edited by H.Wilhelmsson (plenum Press, New York, 1977), p.142.
10. J.Gibbons et al, J. Plasma Phys. 17,pt.2, 153 (1977).
11. J.Hale, Ordinary Differential Equations, (Wiley-Interscience, New York, 1969).
12. J.Z.Wilcox and T.J.Wilcox, Phys. Rev. Lett. 37,1160 (1975).